Topological Robotics

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Ever since the literary works of Capek and Asimov, mankind has been fascinated by the idea of robots. Modern research in robotics reveals that, along with many other branches of mathematics, topology has a fundamental role to play in making these grand ideas a reality. This minicourse will be an introduction to topological robotics – a new discipline situated on the crossroads of topology, engineering and computer science. Currently topological robotics has two main streams: firstly, studying pure topological problems inspired by robotics and engineering and, secondly, applying topological ideas, topological language, topological philosophy and developed tools of algebraic topology to solve specific problems of engineering and computer science. In the course I will discuss the following topics:

1. Configuration Spaces of Mechanical Linkages

Configuration spaces of linkages represent a remarkable class of closed smooth manifolds, also known as polygon spaces. I will show how Morse theory techniques can be used to compute Betti numbers of these manifolds. I will describe solution of Walker's conjecture - a full classification of manifolds of linkages in terms of combinatorics of chambers and strata determined by a collection of hyperplanes in $\mathbf{R}^n$. In many applications (such as molecular biology and statistical shape theory) the lengths of the bars of a linkage are known only approximately; this explain why one wants to study mathematical expectations of topological invariants of varieties of linkages. I will describe some recent results expressing asymptotic values of the average Betti numbers of polygon spaces when the number of links n tends to infinity.

2. Topology of Robot Motion Planning

The motion planning problem of robotics leads to an interesting homotopy invariant $\mathbf{TC}(X)$ of topological spaces which measures the "navigational complexity" of X, viewed as the configuration space of a system. $\mathbf{TC}(X)$ is a purely topological measure of how difficult it is to perform path-planning on a configuration space which is continuous with respect to endpoints. The computation of this complexity provides a subtle topological problem inspired by physical systems. I will give an account of main properties of $\mathbf{TC}(X)$ and will explain how one can compute $\mathbf{TC}(X)$ using cohomology algebra of X and action of cohomology operations. I will also mention certain specific motion planning problems, for example the problem of coordinated collision free control of many particles.