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The Peirce decomposition for generalized Jordan triple systems of finite order
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Review text:

A vector space J over a field ϕ equipped with a triple product (xyz) is called a generalized Jordan triple system (J.t.s.) if

$$(ab(cdf)) = ((abc)df) - (c(bad)f) + (cd(abf)), \quad \forall a,b,c,d,f \in J.$$

Every generalized J.t.s. can be embedded, by a funtorial construction, in a $\mathbb{Z}$-graded Lie algebra (possibly infinite dimensional) $\mathcal{L}(J) = \sum_{-\infty}^{\infty} V_i$ with involution τ such that $\tau(V_{-1}) = V_1$. Moreover,

(1) $[V_1, V_{-1}] = V_0$,

(2) $[V_i, V_1] = V_{i+1}$ and $[V_{-i}, V_{-1}] = V_{-(i+1)}$ for every $i \in \mathbb{N}$.

Conversely, every $\mathbb{Z}$-graded Lie algebra $L = \sum_{-\infty}^{\infty} L_i$ with involution τ such that $\tau(L_{-1}) = L_1$ gives rise to a generalized J.t.s. L_1 with product $(a,b,c) = [[a,\tau(b)],c)]$.

A generalized J.t.s. J is said to have finite order if the corresponding graded Lie algebra $\mathcal{L}(J)$ has a finite number of nonzero components. Moreover, J has order l if the graded Lie algebra $\mathcal{L}(J)$ has the form $\mathcal{L}(J) = \bigoplus_{i=-l}^{l} L_i$ where at least one of V_l or V_{-l} is nonzero. An element of a generalized J.t.s. is a tripotent if $(e,e,e) = e$.

In the paper under review the authors prove that every tripotent of a generalized J.t.s. J of order l, over a field of characteristic 0 or bigger than $2l+1$, induces a decomposition of J into a direct sum of $l^2 + 2l$ components, such that each component consists of simultaneous eigenvectors of the linear operators $L : J \to J$ for $L(x) = (e,e,x)$ and $R : J \to J$ for $R(x) = (x,e,e)$. This decomposition generalizes the usual Pierce decomposition obtained when J is a J.t.s., and can be interesting as a first step in determining the structure of generalized J.t.s in terms of tripotents.

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