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Author:
Loos, Ottmar; Petersson, Holger P.; Racine, Michel L.

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Inner derivations of alternative algebras over commutative rings

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Review Text:
In [R. D. Schafer, Inner derivations of non-associative algebras, Bull. Amer. Math. Soc. 55 (1949), 769--776.], R.D. Schafer introduced a notion of inner derivations for arbitrary non-associative algebras over fields. This new notion reduces to the usual one when dealing with Lie or unital associative (resp. Jordan) algebras. Moreover, inner derivations in this sense always form an ideal in the full derivation algebra. Unfortunately, inner derivations in the sense of Schafer do not satisfy the Mapping Principle even for the class of alternative algebras (the Mapping Principle states that for any homomorphism $f : A \to B$ of algebras A and B and any inner derivation D of A, there exists an inner derivation D' of B with is f-related to D, i.e., $f(D(a))=D'(f(a))$ for all $a\in A$). In view of this result, E. Neher suggested to come back to the old idea of defining individually inner derivations for each class of relevant non-associative algebras, and the authors of this paper devote this work to study a suitable notion of inner derivations for alternative algebras.

In section 1 the authors introduce for a general non-associative algebra A what they call the Lie multiplication derivation algebra

of A, $LMDer(A)$, which is a particular subset of derivations of A contained in the Lie multiplication algebra of A. This set is
an ideal of the algebra of derivations $Der(A)$ of A and it coincides with the usual set of inner derivations for associative and linear Jordan algebras, and with the set of inner derivations in
the sense of Schafer if A is a general non-associative unital algebra. In the rest of this section the authors study the well behavior of $LMDer(A)$ under suitable scalar extensions.

In section 2, following A5.2 of [K. McCrimmon, Alternative algebras, http://www.
mathstat.uottawa.ca/$\sim$ neher/Papers/alternative/, 1980.] the authors describe the Lie multiplication derivation algebra $LMDer(A)$ of an alternative algebra A. The elements of $LMDer(A)$
do not satisfy the Mapping Principle, so they introduce the more restrictive set
of inner derivations $InDer_{Alt}(A)$ of A. This set is an ideal
of $Der(A)$ and its elements do satisfy the Mapping Principle. Among
all inner derivations of A, the authors highlight three types of
inner derivations: associator derivations, which are zero if A is
associative and play an important role for Octonion algebras, standard derivations, which are zero if A is commutative, and commutator derivations, which only appear in algebras with 3-torsion. Moreover, the sets of these three types of inner derivations are ideals of $Der(A)$ and commute with flat base changes provided the algebra itself is finitely spanned as a k-module.

In Section 3, the authors introduce the notion of derivation functor and show that the ideals of standard, associator and commutator derivations are all induced by suitable derivation functors. These derivation functors commute with flat base changes and in the particular case of the standard derivation functor, it even commutes with arbitrary base changes without any finiteness assumptions on the underlying algebra.

Section 4 is used to study Octonion algebras over commutative rings,
and the results of this section are applied in section 5 to show that the only proper inner derivations in Octonion algebras are associator derivations.

Remarks: