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Alternative elements in the Cayley-Dickson algebras. (English)

García-Compeán, Hugo (ed.) et al., Topics in mathematical physics, general relativity and cosmology in honor of Jerzy Plebański. Proceedings of the 2002 international conference, Cinvestay, Mexico City, Mexico, September 17–20, 2002. Hackensack, NJ: World Scientific. 333-346 (2006).

The Cayley-Dickson construction starts with an arbitrary unital algebra A with involution over a ring of scalars ϕ and an invertible scalar $\mu \in \phi$, and doubles A to get the following unital algebra with involution: $\mathcal{KD}(A, \mu) = A \oplus A$ with product and involution given by

$$(x_1, x_2) * (y_1, y_2) = (x_1 y_1 + \mu \overline{y_2} x_2, y_2 x_1 + x_2 \overline{y_1}), \quad \overline{(x_1, x_2)} = (\overline{x_1}, -x_2).$$

The Cayley-Dickson process consists of iterating the Cayley-Dickson construction over an over again. In the paper under review, the author studies the real algebras $\mathbb{A}_n$ defined by induction by the Cayley-Dickson process as follows: $\mathbb{A}_0 = \mathbb{R}$ and $\mathbb{A}_n = \mathcal{KD}(\mathbb{A}_{n-1}, -1)$. It is well known that $\mathbb{A}_0 = \mathbb{R}$ and $\mathbb{A}_1 = \mathbb{C}$ are associative and commutative, that $\mathbb{A}_2 = \mathbb{H}$, the quaternions, is associative non commutative, and that $\mathbb{A}_3 = \mathbb{O}$, the octonions, is neither associative nor commutative but it is alternative. For $n \geq 4$, $\mathbb{A}_n$ is not even alternative. The main result of the paper characterizes the alternative elements of $\mathbb{A}_n$ (an element $a \in \mathbb{A}_n$ is called alternative if $(a, a, x) = 0$ for all $x \in \mathbb{A}_n$, where $(x, y, z) := (xy)z - x(yz)$ is the associator of $x, y, z \in \mathbb{A}_n$): An element $(x, y) \in \mathbb{A}_n$ $(n \geq 3)$ is alternative if and only if x, y are alternative elements in $\mathbb{A}_{n-1}$ and their pure parts are linearly dependent. The author also studies strongly alternative elements in $\mathbb{A}_n$ ($a \in \mathbb{A}_n$ is strongly alternative if it is alternative and for every $x \in \mathbb{A}_n$, $(a, x, x) = 0$), and gives a criterion to know which elements generate associative subalgebras inside $\mathbb{A}_n$.

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∗17A45 Quadratic algebras

17A75 Composition algebras