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On Bernstein algebras satisfying chain conditions. (English)

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An algebra A over a field K with a nonzero homomorphism $\omega : A \to K$ is called a baric algebra. A Bernstein algebra is a commutative baric algebra which satisfies $(x^2)^2 = \omega(x)^2 x^2$ for every $x \in A$. If, in addition, A satisfies the Jordan identity (for every $x, y \in A$, $x(x^2 y) = x^2(xy)$) A is called a Bernstein-Jordan algebra. The principal result on Bernstein-Jordan algebras in the paper states that a Bernstein-Jordan algebra (A, ω) with $\mathrm{Ker}(\omega)^2$ of finite codimension is finite dimensional.

The above result is the main tool in order to prove that in a Bernstein-Jordan algebra the notions of Artinian, Noetherian, finite generation or finite dimension are equivalent condition (a Bernstein algebra is Artinian – Noetherian – if it satisfies the descending – ascending – chain condition on ideals).

Every Bernstein algebra which nontrivial product contains a nonzero idempotent e and, therefore, an associated Peirce decomposition $A = Ke \oplus U \oplus V$ with $\mathrm{Ker}(\omega) = U \oplus V$, $U = \{u \in A \mid 2eu = u\}$ and $V = \{v \in A \mid ev = 0\}$. In this conditions, the authors introduce a structure of left $K\langle V\rangle$-module over the ideal $\mathrm{ann}_U(U) := \{u \in U \mid uU = 0\}$, where $K\langle V\rangle$ is the associative unital free ring generated by V, and characterize when a Bernstein algebra satisfies any chain condition: A Bernstein algebra $A = Ke \oplus U \oplus V$ is Artinian (resp., Noetherian or finitely generated) if and only if $A/\mathrm{ann}_U(U)$ is finite dimensional and the $K\langle V\rangle$-module $\mathrm{ann}_U(U)$ is Artinian (resp., Noetherian or finitely generated). As a corollary they obtain that every Noetherian Bernstein algebra is finitely generated. They also prove that apart from this implication, these three finiteness conditions are independent of each other.

Finally, they prove that a Noetherian Bernstein algebra is countably dimensional, and relate the Artinian case to an open problem concerning Artinian modules: Every Artinian Bernstein algebra is countably dimensional if and only if every Artinian left module over a finitely generated associative algebra is countably generated.

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