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On simple Filippov superalgebras of type $A(n,n)$

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*Review text:*

An n-ary Leibniz algebra over a field $k$ is a vector space $L$ over $k$ with an n-ary multilinear operation $(x_1, x_2, \dots, x_n)$ satisfying that for every $y_2, \dots, y_n \in L$ the operator of left multiplication $L_{(y_2,\dots,y_n)} : L \to L$ defined by $xL_{(y_2,\dots,y_n)} = (x, y_2, \dots, y_n)$ verifies the identity:

$$(x_1, x_2, \dots, x_n)L_{(y_2,\dots,y_n)} = \sum_{i=1}^{n}(x_1, \dots, x_i L_{(y_2,\dots,y_n)}, \dots, x_n)$$

If this n-ary operation is anti-commutative, it is said that $L$ is a Filippov (n-Lie) algebra over $k$. In [Communications in Algebra, 31 No. 1 (2003) 197-215], A. P. Pojidaev studied finite-dimensional commutative n-ary Leibniz algebras over a field of characteristic 0 and showed that there exist no simple ones. Furthermore, the finite-dimensional simple Filippov algebras over an algebraically closed field of characteristic 0 have been classified in [PhD thesis, Siegen University, 1993] by L. Wuxue. Both types of algebras, commutative and anti-commutative n-ary Leibniz algebras, are particular cases of n-ary Filippov superalgebras: a $\mathbb{Z}_2$-graded n-ary algebra $L = L_0 \oplus L_1$ over a field $k$ with an n-ary multilinear $\mathbb{Z}_2$-graded operation $[x_1, x_2, \dots, x_n]$ that for every family of homogeneous elements $x_1, x_2, \dots, x_n \in L$ satisfies the super-anticommutativity property:

$$[x_1, \dots, x_{i-1}, x_i, \dots, x_n] = (-1)^{p(i-1)p(i)}[x_1, \dots, x_i, x_{i-1}, \dots, x_n]$$

where $p(x_i) = \alpha$ means that $x_i \in L_\alpha$ and the super generalized Jacobi identity: For every $y_2, \dots, y_n \in L$,

$$[[x_1, x_2, \dots, x_n], y_2, \dots, y_n] = \sum_{i=1}^{n}(-1)^{pq_i}[x_1, \dots, [x_i, y_2, \dots, y_n], \dots, x_n]$$

where $p = \sum_{i=1}^{n} p(y_i)$ and $q_i = \sum_{j=i+1}^{n} p(x_j)$.

In [Algebra Logika, 47 No. 2 (2008) 2610-261] A. P. Pojidaev, studied Filippov superalgebras of type $B(n,m)$ (a Filippov superalgebra $L$ is of type $G$ if $Inder(L) \cong G$ for $G$ a Lie superalgebra where $Inder(L)$ denotes the linear space generated by the strictly inner derivations of $L$). In the paper under review the authors prove that there exist no simple finite dimensional Filippov superalgebras of type $A(n,n)$.

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