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Benkart, Georgia; Fernández López, Antonio
The Lie inner ideal structure of associative rings revisited
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Keywords: Lie inner ideals; Skew-symmetric elements.

Review text:
In this paper the authors complete the description of the Lie inner ideal structure of simple Artinian rings with involution and of simple rings with involution and minimal one-sided ideals. An inner ideal of a Lie algebras L is a submodule B of L such that $[B,[B,L]] \subset B$. Inner ideals are the analogues of one-sided ideals in associative rings and algebras.
The authors study Lie algebras of the form $L = [K,K]/(Z \cap [K,K])$ where $K = \text{Skew}\ (R,*)$ is the set of all skew elements of a simple associative ring R with involution $*$ (of characteristic not 2 or 3), with minimal one-sided ideals, and Z is the center of R. The description of the inner ideals of such Lie algebras L was begun in a paper of G. Benkart [The Lie inner ideal structure of associative rings. J. Algebra 43 (1976) 561-584], but there was a case missing from that paper: when the inner ideal B is such that $b^2 = 0$ for all $b \in B$ and B cannot be written in the form eKe for any idempotent $e \in R$ with $e^*e = 0$. These missing abelian inner ideals are indeed point spaces (abelian inner ideals all of whose elements x satisfy $[x,[x,L]] = \mathbb{F}x$), and can be found by using the strong connection between abelian inner ideals of Lie algebras and inner ideals of some Jordan pairs called subquotients that are associated to those Lie inner ideals (see [A construction of gradings of Lie algebras. Int. Math. Res. Not. 16, (2007) Art.ID rnm051, 34 pp]).

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