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Review text:

Let A and B be rings. A linear map $\theta : A \to B$ is called zero-product preserving if it has the property that $\theta(a)\theta(a') = 0$ whenever $aa' = 0$, $a, a' \in A$. In [Mappings preserving zero products. Studia Math. 155(1)(2003), 77-94] and [Maps characterized by action on zero products. Pacific J. Math. 216(2) (2004), 217-228] Chebotar et al. characterized, under certain conditions, that zero-product preserving maps are of the form $\theta(x) = \lambda f(x)$ with $\lambda \in \mathcal{C}(B)$, the extended centroid of B, and $f : A \to B$ a homomorphism of rings.

In the paper under review the author studies similar problems for prime rings with involution. So, he proves that if A is a prime ring with involution which is generated by idempotents, and $\theta : A \to A$ is a bijective additive map such that $\theta(x)\theta(y)^* = 0$ whenever $xy^* = 0$, then there is a $*$-monomorphism $g : A \to \mathcal{Q}$ such that $\theta(xy) = \theta(x)g(y)$ for all $x, y \in A$. Moreover, he can weaken the (strong) condition of A to be generated by idempotents proving: Let A be a prime ring with involution, containing a nontrivial idempotent and suppose that $\theta : A \to A$ is an additive bijection that satisfies both:

(i) $\theta(x)\theta(y)^* = 0$ whenever $xy^* = 0$; and

(ii) $\theta(x)^*\theta(y) = 0$ whenever $x^*y = 0$.

Then there exist a $*$-monomorphism $g : A \to \mathcal{Q}$ and an element $t \in \mathcal{Q}$ with $tt^* \in C$ such that $\theta(x) = tg(x)$ for all $x \in A$.

An important tool in the proof of these theorems is the use of the theory of funcional identities given in the book of M. Brešar, M. A. Chebotar, and W. S. Martindale, 3rd [Functional Identities. Basel-Boston-Berlin: Birkhauser Verlag (2007)].