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Jain, S. K. [Jain, Surender Kumar] (1-OH); **Al-Hazmi, Husain S.** (1-OH);
Alahmadi, Adel N. (1-OH)

Right-left symmetry of right nonsingular right max-min CS prime rings. (English summary)

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A submodule N of an R -module M is called closed in M if it has no proper essential extensions in M . A right ideal of R is closed if it is a closed R_R -submodule. A ring R is called right CS if every closed right ideal is a direct summand of R and it is called right max (min) CS if only the maximal (minimal) closed right ideals are direct summands of R . Similarly, we can define the symmetric left notions.

In 2000, Dinh Van Huynh, S. K. Jain and S. R. López-Permouth [Proc. Amer. Math. Soc. **128** (2000), no. 11, 3153–3157; [MR1670375 \(2001b:16021\)](#)] proved that a prime right Goldie ring which is right CS of at least right uniform dimension 2 is left Goldie and left CS. In this paper the authors study similar symmetries in a more general setting: Let R be a prime ring which is right nonsingular with a right uniform ideal. Then,

- (a) right min CS implies (left) nonsingularity,
- (b) left nonsingularity and right max CS imply left min CS, and
- (c) right CS implies left min CS.

Moreover, they obtain the nice (left-right) symmetry: If R is a prime ring which is not a domain, then R is right nonsingular and right max-min CS with a uniform right ideal if and only if R is left nonsingular and left max-min CS with a uniform left ideal.

Reviewed by *Miguel A. Gómez Lozano*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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