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On strong Goldie dimension. (English summary)
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A module M has Goldie dimension n (written $\mathrm{G}\dim M=n$) if it contains an essential submodule which is a direct sum of n uniform submodules (where a submodule N of M is uniform if it is nonzero and for any nonzero submodules N_1 and N_2 of N, $N_1\cap N_2\neq 0$). In these conditions, any direct sum of nonzero submodules of M contains at most n summands. Following the notions of right global dimension or right weak dimension of a ring, the authors introduce the notion of strong Goldie dimension of a module: A module M is said to have strong Goldie dimension equal to n ($\mathrm{SG}\dim M=n$) if $\sup\{\mathrm{G}\dim M/N\mid N\leq M\}=n$. In this case M is called strongly finite dimensional. A ring R is called right (left) strongly finite dimensional if it is strongly finite dimensional as a right (left) R-module.

In the paper under review the authors characterize the strong Goldie dimension of a module M as

$$\mathrm{SG}\dim M=\sup\{\mathrm{codim}\, N,\ N\leq M\}$$

and prove that strong finite dimensionality is a Morita invariant property: If R is a unital ring and $S=M_n(R)$, then

$$\mathrm{G}\dim S=n\mathrm{G}\dim R.$$

Moreover, they characterize some classes of strongly finite dimensional modules: The modules with strong Goldie dimension equal to 1 are the uniserial modules (a module M is uniserial if the lattice of its submodules is totally ordered by inclusion). The right (left) Artinian rings are the right (left) semi-Artinian rings with right (left) strongly finite dimension (where a ring R is right (left) semi-Artinian if every right (left) R-module has an essential right (left) socle). As a consequence, strong finite dimensionality is not a left-right symmetric property.

Reviewed by *Miguel A. Gómez Lozano*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.