A module $M$ has Goldie dimension $n$ (written $G \dim M = n$) if it contains an essential submodule which is a direct sum of $n$ uniform submodules (where a submodule $N$ of $M$ is uniform if it is nonzero and for any nonzero submodules $N_1$ and $N_2$ of $N$, $N_1 \cap N_2 \neq 0$). In these conditions, any direct sum of nonzero submodules of $M$ contains at most $n$ summands. Following the notions of right global dimension or right weak dimension of a ring, the authors introduce the notion of strong Goldie dimension of a module: A module $M$ is said to have strong Goldie dimension equal to $n$ ($SG \dim M = n$) if $\sup \{G \dim M/N \mid N \leq M\} = n$. In this case $M$ is called strongly finite dimensional. A ring $R$ is called right (left) strongly finite dimensional if it is strongly finite dimensional as a right (left) $R$-module.

In the paper under review the authors characterize the strong Goldie dimension of a module $M$ as

$$SG \dim M = \sup \{\text{codim } N, N \leq M\}$$

and prove that strong finite dimensionality is a Morita invariant property: If $R$ is a unital ring and $S = M_n(R)$, then

$$G \dim S = nG \dim R.$$ 

Moreover, they characterize some classes of strongly finite dimensional modules: The modules with strong Goldie dimension equal to 1 are the uniserial modules (a module $M$ is uniserial if the lattice of its submodules is totally ordered by inclusion). The right (left) Artinian rings are the right (left) semi-Artinian rings with right (left) strongly finite dimension (where a ring $R$ is right (left) semi-Artinian if every right (left) $R$-module has an essential right (left) socle). As a consequence, strong finite dimensionality is not a left-right symmetric property.

Reviewed by Miguel A. Gómez Lozano

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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