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The nucleus of the free alternative algebra. (English summary)

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The defining axioms for an alternative algebra A are the left and right alternative laws:

$$(x, x, y) = 0 = (y, x, x),$$

where $(x, y, z) := (xy)z - x(yz)$ is the associator of $x, y, z \in A$. Consequently, in an alternative algebra, the associator is an alternating function of its arguments. Therefore, the nucleus of an alternative algebra A is the set

$$N(A) = \{p \in A |\ (p, x, y) = 0 \quad \forall\, x, y \in A\}.$$

In this paper the authors determine, using computer procedures, a basis for the elements of degree 5 in the nucleus of a free alternative algebra A over the field $\mathbb{Z}/103\mathbb{Z}$ (they work with the prime number 103 because it is only one byte long and it is bigger than the degree of any identity that appears). In any case, the method they use is valid for characteristic zero or a large-enough prime). Moreover, they prove that there are no elements of lower degree (less than 5) in $N(A)$.

Reviewed by *Miguel A. Gómez Lozano*

References

1. J. M. Clifton. "A Simplification of the Computation of the Natural Representation of the Symmetric Group S_n." *Proc. Amer. Math. Soc.* 83:2 (1981), 248–250. MR0624907 (82j:20024)
2. V. T. Filippov. "Malcev Algebras Satisfying Engel's Condition." *Algebra and Logic* 14:4 (1975), 271–281. MR0506471 (58 #22188)
3. I. R. Hentzel. "Processing Identities by Group Representation." In *Computers in Nonassociative Rings and Algebras*, edited by E. E. Beck and B. Kolman, pp. 13–40. New York: Academic Press, 1977. MR0463251 (57 #3204)
4. I. R. Hentzel and D. P. Jacobs. "A Dynamic Programming Method for Building Free Algebras." *Comput. Math. Appl.* 22:12 (1991), 61–66. MR1140047 (92m:17006)
5. I. R. Hentzel and L. A. Peresi. "Identities of Cayley-Dickson Algebras." *J. Algebra* 188:1 (1997), 292–309. MR1432358 (98a:17004)
6. I. R. Hentzel and L. A. Peresi. "A Nonzero Element of Degree 7 in the Center of the Free Alternative Algebra." *Comm. Alg.* 31:3 (2003), 1279–1299. MR1971063 (2004e:17028)
7. I. R. Hentzel and L. A. Peresi. "Central Elements of Minimal Degree in the Free Alternative Algebra." In *Non-Associative Algebra and Its Applications*, edited by L. Sabinin, L. V. Sbitneva, I. P. Shestakov, pp. 195–204, *Lecture Notes in Pure and Appl. Math.* 246, Chapman & Hall/CRC, 2006. MR2203706 (2006i:17046)
8. D. P. Jacobs, D. Lee, S. V. Muddana, A. J. Offutt, K. Prabhu, and T. White-

ley. *Version 3.0. Albert's User Guide.* Clemson University, 1996. Available online (www.cs.clemson.edu/∼dpj/albertstuff/albert.html).
9. G. James and A. Kerber. *The Representation Theory of the Symmetric Group.* Encyclopedia of Mathematics and Its Applications, 16. Reading: Addison-Wesley, 1981. MR0644144 (83k:20003)
10. E. Kleinfeld. "Simple Alternative Rings." *Ann. of Math. (2)* 58 (1953), 544–547. MR0058581 (15,392b)
11. E. Kleinfeld. "Right Alternative Rings." *Proc. Amer. Math. Soc.* 4 (1953), 939–944. MR0059888 (15,595j)
12. A. I. Malcev. "On Algebras Defined by Identities." *Mat. Sbornik N. S.* 26 (68) (1950), 19–33. MR0033280 (11,414d)
13. J. McKay. "The Largest Degrees of Irreducible Characters of the Symmetric Group." *Math. Comp.* 30 (1976), 624–631. MR0404414 (53 #8216)
14. A. Regev. "On the Codimensions of Matrix Algebras." In *Algebra—Some Current Trends (Varna, 1986)*, pp. 162–172, Lecture Notes in Math. 1352. Berlin: Springer, 1988. MR0981825 (89k:16041)
15. I. P. Shestakov. "Free Malcev Superalgebra on One Odd Generator." *J. Algebra Appl.* 2:4 (2003), 451–461. MR2020951 (2004h:17031)
16. I. Shestakov and N. Zhukavets. "The Universal Multiplicative Envelope of the Free Malcev Superalgebra on One Odd Generator." *Comm. Alg.* 34:4 (2006), 1319–1344. MR2220815 (2007b:17051)
17. I. Shestakov and N. Zhukavets. "Speciality of Malcev Superalgebras on One Odd Generator." *J. Algebra* 301:2 (2006), 587–600. MR2236758 (2007b:17052)
18. W. Specht. "Gesetze in Ringen I." *Math. Z.* 52 (1950), 557–589. MR0035274 (11,711i)
19. K. A. Zhevlakov, A. M. Slin'ko, I. P. Shestakov, and A. I. Shirshov. *Rings That Are Nearly Associative.* New York: Academic Press, 1982. MR0668355 (83i:17001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.