DE055579878
Vaš, Lia
Extending higher derivations to rings and modules of quotients
Int. J. Algebra 2, No. 13-16, 711-731 (2008).

MSC Classification: 16S90 16W25

Keywords: Derivation; Higher Derivation, Ring of Quotients; Module of Quotients; Torsion Theory.

Review text:
A derivation on a ring R is an additive map $\delta : R \to R$ such that $\delta(rs) = \delta(r)s + r\delta(s)$ for all $r, s \in R$, and if M is an R-module, a δ-derivation on M is an additive map $d : M \to M$ such that $d(mr) = d(m)r + m\delta(r)$. In [J. S. Golan, Extensions of derivations to modules of quotients, Communications in Algebra, 9 (1981), no. 3, 275–281] the author proved that if δ is a derivation on R, M is a right R-module, d is a δ-derivation on M and $\tau = (\mathcal{T}, \mathcal{F})$ is a hereditary torsion theory, then,
(1) if M is torsion-free, d extends to a derivation on the module of quotients $M_{\mathcal{F}}$ such that $dq_M = q_M d$;
(2) if $d(\mathcal{T}(M)) \subset \mathcal{T}(M)$, d extends to a derivation on the module of quotients $M_{\mathcal{F}}$ such that $dq_M = q_M d$.
Moreover, this extension is unique by [P. E. Bland, Differential torsion theory, Journal of Pure and Applied Algebra, 204 (2006), 1–8].
In the paper under review the author goes over some results on [P. E. Bland, Higher derivations on rings and modules. Int. J. Math. Math. Sci., (2005), no. 15, 2373-2387] and [S. H. Rim, Extensions of higher anti-derivations to modules of quotients. J. Korean Math. Soc., 24 (1987), no. 1, 25-31] and studies new and equivalent conditions under which one can extend a higher derivation from a module to the module of quotients (recall that a higher derivation of order n on a ring R is an indexed family $\{\delta_i\}_{i=0}^n$ of additive maps δ_i such that δ_0 is the identity map on R and $\delta_i(rs) = \sum_{j=0}^{i} \delta_j(r)\delta_{i-j}(s)$ for all $i \leq n$; an indexed family $\Delta = \{\delta_n\}_{n\in\omega}$ is a higher derivation on R if $\{\delta_i\}_{i=1}^n$ is a higher derivation of order n for all $n \in \omega$). Moreover, he proves that some important torsion theories are higher derivations: The Lambek torsion theory, the Goldie torsion theory or any perfect torsion theory, which include the classical torsion theory, are higher differential. The paper finishes with some interesting open questions on this matter.

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