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A class of nonassociative algebras arising from quadratic ODEs. (English)

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http://dx.doi.org/10.1081/AGB-200051139
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In [*L. Markus*, Ann. Math. Stud. 45, 185–213 (1960; Zbl 0119.29803)] associated to every quadratic ODE a (possibly non-associative) commutative algebra and studied the solutions of quadratic ODEs via this algebra: Every system of homogeneous quadratic differential equations $\overrightarrow{x}' = K(\overrightarrow{x})$ in $\mathbb{R}^n$, where $K : \mathbb{R}^n \to \mathbb{R}^n$ is a quadratic form, defines an algebra structure on the vector space $\mathbb{R}^n$ given by the product $\overrightarrow{x} * \overrightarrow{y} := B(\overrightarrow{x}, \overrightarrow{y})$, where $B : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the symmetric bilinear form associated to K. In the paper under review the authors classify the three-dimensional real commutative algebras associated, via the Markus construction, to homogeneous quadratic systems of ODEs in $\mathbb{R}^3$ with a plane of critical points, proving the following result: A three-dimensional real commutative algebra corresponds to a homogeneous quadratic system of ODEs which contains a plane of critical points if and only if there exists a basis $\{N_1, N_2, E\}$ in which the algebra multiplication table is given by

$$\begin{array}{lll} N_1N_1 = 0 & N_1N_2 = 0 & N_1E = aN_1 + bN_2 + cE \\ N_2N_1 = 0 & N_2N_2 = 0 & N_2E = \alpha N_1 + \beta N_2 + \gamma E \\ EN_1 = aN_1 + bN_2 + cE & EN_2 = \alpha N_1 + \beta N_2 + \gamma E & EE = dN_1 + eN_2 + fE. \end{array}$$

Moreover, they classify these algebras up to isomorphism and give a simple multiplication table for each of them. This classification has been already used in [*M. Mencinger*, Nonlinearity 16, 201–218 (2003; Zbl 1030.34047)] in order to study which homogeneous quadratic systems of ODEs with a plane of critical points has stable origin.

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Keywords : Algebra classification; Autonomous differential equations; Riccati differential equations; Stability of critical points
Classification :
∗17A60 Structure theory of general nonassociative rings and algebras
34C20 Transformation of ODE and systems
34D20 Lyapunov stability of ODE