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Kim, Hyuk; Kim, Kyunghee

The structure of assosymmetric algebras. (English)

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In the paper under review the authors study the structure of finite dimensional assosymmetric algebras over $\mathbb{R}$ or $\mathbb{C}$ (an algebra A is called assosymmetric if the associator $(x, y, z) := (x \cdot y) \cdot z - x \cdot (y \cdot z)$ remains invariant under each permutation of x, y, z). Every associative algebra is assosymmetric. Moreover, E. Kleinfeld proved that a semiprime assosymmetric algebra is associative [see *E. Kleinfeld*, Proc. Am. Math. Soc. 8, 983–986 (1957; Zbl 0082.02903)]. An assosymmetric algebra becomes a Lie algebra under the skew-symmetric product $[x, y] = x.y - y.x$ and, as in the Lie setting, the right multiplication $\rho_b(a) = a.b$ gives rise to a symmetric bilinear form $\sigma(x, y) := \mathrm{tr}\rho_{x.y}$ which plays a similar role to the Killing form in Lie algebras.
In the first part of the paper, the authors study the Koszul radical of A (which coincides with the maximal left ideal of A contained in $\mathrm{Ker}\,\mathrm{tr}\,\rho$, by a result of Helmstetter). They prove that this radical is the ideal $A^{\perp}$ with respect to σ, coincides with the transitive radical introduced in [*C. Bai* and *D. Meng*, Commun. Algebra 28, No. 6, 2717–2734 (2000; Zbl 0966.17001)], with the solvable radical and with the nilpotent radical of A, and it is the maximal complete ideal of A. These characterizations make possible to prove the main theorem of the paper, the Wedderburn decomposition: Let A be an assosymmetric algebra over $\mathbb{C}$ such that $A/R(A)$ has no 1-dimensional factors. Then there exists a subalgebra section of $A/R(A)$ in A complementary to $R(A)$, and any two such complementary subalgebras are transformed into each other by a product of automorphisms of the form $e^{\mathrm{ad}(r)}$ where $r \in R(A)$. For the case of real assosymmetric algebras, the same conclusion holds if the complexification of $A/R(A)$ has no one-dimensional complex factors.

Miguel Angel Gomez Lozano (Malaga, Spain)

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17A60 Structure theory of general nonassociative rings and algebras