Arch. Math. 70 (1998) 219–227
0003-889X/98/030219-09 $ 3.30/0

Archiv der Mathematik

Local-to-global inheritance of primitivity in Jordan algebras

By

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Abstract. We show that a strongly prime Jordan algebra having a primitive local algebra is primitive. As a tool, similar results concerning one-sided primitivity and $*$-primitivity of associative algebras are established.

Introduction. The notion of local algebra of a Jordan system, introduced in [17], has proved to be a basic tool in the study of primitivity [3, 4, 7]. Unlike the algebra case, primitivity of a Jordan pair is considered *at a particular element*, and shown to be equivalent to primitivity of its local algebra at this element, under the assumption of strong primeness [4]. As a consequence, the results on local inheritance of primitivity in Jordan algebras [9] were an important tool to have some possibility of changing the element at which primitivity holds, which is essential in the description of primitive Jordan pairs and triple systems given in [4].

In this paper we show the inheritance of primitivity by a strongly prime Jordan algebra from any primitive local algebra. Apart from its intrinsic interest, this result will play an important role in the definite solution of the above problem on the "location" of primitivity for Jordan systems [5], as one could expect.

The main ingredients of the proofs are the general properties of primitivity and its local inheritance in Jordan algebras [8, 9], as well as some facts concerning primitive pairs and triple systems [3, 4]. After a preliminary section, collecting known results which will be used in the sequel, we begin in Section 1 with the study of local-to-global inheritance of one-sided primitivity and $*$-primitivity in associative algebras. The corresponding result for Jordan algebras splits into two cases: the hermitian case and the "finite" case. Due to the fact that the initial object is the local algebra, the above two cases are those of non-homotope-PI and homotope-PI Jordan algebras. This forces us to use the theory of pairs and triple systems, so that results on the relationship of a Jordan algebra and its underlying triple system are needed. Non-homotope-PI Jordan algebras are studied in Section 2 using the main result of Section 1, while the homotope-PI case in Section 3 follows directly by an argument involving Peirce decompositions and the socle. We unify the different cases in the main theorem of Section 4, getting also an important consequence on inheritance of primitivity from Peirce components.

Mathematics Subject Classification (1991): 17C10, 17C27.

*) Partially supported by the DGICYT, PB94-1311-C03 and the II Plan Regional de Investigación del Principado de Asturias, FICYT PB-PGI97-02.

0. Preliminaries.

0.1. We will deal with associative and Jordan algebras over an arbitrary ring of scalars Φ. The reader is referred to [8, 9, 12, 16] for basic results, notation and terminology.

0.2. Algebras and triple systems. Due to the nature of the results, we will be led to consider associative and Jordan triple systems. For general facts, notation and terminology, see [2, 3, 4, 12]. We will mainly deal with triple systems coming from algebras:

– If R is an associative algebra with product denoted by juxtaposition xy, for any $x, y \in R$, we can consider the associative triple system structure on R given by the product xyz, for any $x, y, z \in R$.

– If J is a Jordan algebra with products $x^2, U_x y$, for any $x, y \in J$, one can consider a Jordan triple system structure on J given by $P_x y = U_x y$ for any $x, y \in J$.

Notice that ideals of an algebra are always ideals of the underlying triple system, while the converse is obviously false [in the algebra of polynomials in a single variable $R = \Phi[X]$, the Φ-module spanned by $\{X^n \mid n \text{ is odd}\}$ is an ideal in the triple sense but not an algebra ideal]. However, semiprimeness, primeness and $*$-primeness coincide for an associative algebra and its underlying triple system (cf. [2]). With respect to Jordan algebras, we have an obvious coincidence of nondegeneracy of the algebra and the underlying triple system, while strong primeness can be handled using a "tight unital hull argument" (cf. [6, 1.12]): since ideals in the triple and algebra senses coincide when the algebra is unital, we transfer strong primeness from a Jordan algebra to a tight unital hull which is then strongly prime as an algebra and as a triple system; this provides strong primeness of the original system by [14, 2.5], since it is an ideal of the unital hull.

0.3. Primitive Jordan algebras are defined in [11] following the corresponding notions of primitivity and $*$-primitivity of associative algebras in terms of modular one-sided ideals [7, Section 0; 18, p. 306]. Thus a Jordan algebra J is primitive if there exits a proper inner ideal K ($K \neq J$), called a *primitizer* of J such that (cf. [8, 0.5, 0.6]):

(i) K is e-modular for some modulus $e \in J$ [11, 2.2, 2.3, 2.4].

(ii) $I + K = J$ for any nonzero ideal I of J.

0.4. One can obtain Jordan systems from associative systems by symmetrization: If R is an associative algebra, we can obtain a Jordan algebra denoted by $R^{(+)}$, over the same Φ-module, with products built out of the associative product by $x^2 = xx$, $U_x y = xyx$, for any $x, y \in R$. Similarly, a Jordan triple system $R^{(+)}$ can be obtained from an associative triple system R by defining $P_x y = xyx$, for any $x, y \in R$.

A Jordan system (algebra or triple system) is said to be *special* if it is a subsystem of $R^{(+)}$ for some associative system R.

0.5. A particularly important example of special Jordan systems are ample subspaces or subsystems of associative systems with involution:

– If R is an associative algebra with involution $*$, a Φ-submodule H contained in the set of symmetric elements $H(R, *)$ is said to be an *ample subspace* of R if it contains all traces and norms of the elements of R ($x + x^*, xx^* \in H$ for any $x \in R$) and $xHx^* \subseteq H$ for any $x \in R$ [13, p. 387; 16, 0.8']. Obviously, H is a subalgebra of $R^{(+)}$.

– If R is an associative triple system with involution $*$ ($* : R \longrightarrow R$ is a linear map of period two, reversing products: $(xyz)^* = z^*y^*x^*$), an *ample subsystem* H is a submodule of symmetric elements ($H \subseteq H(R,*)$) containing all traces $x + x^*$ of the elements of R and satisfying $xHx^* \subseteq H$, for any $x \in R$. Obviously, H is a subsystem of $R^{(+)}$.

0.6. Local algebras of Jordan systems are introduced in [17] generalizing the corresponding notion for associative systems:

– Given an associative triple system R, the *homotope* $R^{(a)}$ of R at $a \in R$ is the associative algebra over the same Φ-module as R with product $x \cdot_a y = xay$, for any $x, y \in R$. The subset $\operatorname{Ker} a = \operatorname{Ker}_R a = \{x \in R \mid axa = 0\}$ is an ideal of $R^{(a)}$ and the quotient $R_a = R^{(a)}/\operatorname{Ker} a$ is called the *local algebra of R at a.*

– Given a Jordan triple system J, the *homotope* $J^{(a)}$ of J at $a \in J$ is the Jordan algebra over the same Φ-module as J with products $x^{(2,a)} = x^2 = P_x a$, $U_x^{(a)} y = U_x y = P_x P_a y$, for any $x, y \in J$. The subset $\operatorname{Ker} a = \operatorname{Ker}_J a = \{x \in R \mid P_a x = P_a P_x a = 0\}$ is an ideal of $J^{(a)}$ and the quotient $J_a = J^{(a)}/\operatorname{Ker} a$ is called the *local algebra of J at a.* When J is nondegenerate, $\operatorname{Ker} a = \{x \in J \mid P_a x = 0\}$.

For an associative or Jordan algebra, local algebras are given by the above definitions, applied to its underlying triple system.

0.7. Global-to-local inheritance in Jordan and associative algebras. Basic results on local algebras of associative and Jordan algebras can be found in [9]. We stress the following local inheritances [9, 0.1, 4.1], some of whose converses are the main results of this paper:

(I) Let R be an associative algebra (resp. an associative algebra with involution $*$), and let $0 \neq a \in R$ (resp. $0 \neq a \in H(R,*)$).

- (i) If R is semiprime, then R_a is semiprime.
- (ii) If R is prime (resp. $*$-prime), then R_a is prime (resp. $*$-prime).
- (iii) If R is left (right) primitive (resp. $*$-primitive), then R_a is left (right) primitive (resp. $*$-primitive).

(II) Let J be a Jordan algebra, $0 \neq a \in J$.

- (i) If J is nondegenerate, then J_a is nondegenerate.
- (ii) If J is strongly prime, then J_a is strongly prime.
- (iii) If J is primitive, then J_a is primitive.

1. Associative algebras. We will begin with the associative version of the main result of the paper. Its proof is much simpler than in the Jordan case, due to the linearity of the notion of one-sided ideal, which allows a direct construction of primitizers. Apart from its independent interest, the inheritance of $*$-primitivity will be explicitly needed in the sequel.

1.1. Theorem. *Let R be a prime (resp. $*$-prime) associative algebra (resp. $*$-algebra) such that R_a is left (right) primitive (resp. $*$-primitive) for some $0 \neq a \in R$ (resp. $0 \neq a \in H(R,*)$). Then R is primitive (resp. $*$-primitive).*

Proof. Assume, for example, that $\bar{R} = R_a$ is left primitive (resp. $*$-primitive) and let $\bar{K}$ be a primitizer (resp. $*$-primitizer) of $\bar{R}$. It is clear that $\bar{K} = K/\mathrm{Ker}\, a$, for some proper left ideal K of $R^{(a)}$. For any modulus $\bar{e} = e + \mathrm{Ker}\, a$ of $\bar{K}$, $R(1 - ae) = \{x - xae \mid x \in R\}$ is a left ideal of R contained in K by $\bar{e}$-modularity of $\bar{K}$.

Let L be the sum of $R(1 - ae)$ for all $e \in R$ such that $\bar{e} = e + \mathrm{Ker}\, a$ is a modulus for $\bar{K}$. Thus we have that

(i) L is a proper left ideal of R

since it is contained in K. Moreover, $R(1 - ae) \subseteq L$ reads

(ii) L is ae-modular for any e such that $\bar{e}$ is a modulus for $\bar{K}$.

Let I be a nonzero ideal (resp. $*$-ideal) of R. Clearly

$$\bar{I} = (I + \mathrm{Ker}\, a)/\mathrm{Ker}\, a$$

is an ideal of $\bar{R}$. Moreover $\bar{I} \neq 0$ since otherwise $I \subseteq \mathrm{Ker}\, a$, i.e., $aIa = 0$, which is impossible by primeness (resp. $*$-primeness) of R. Therefore $\bar{I} + \bar{K} = \bar{R}$ and it is possible to find $\bar{e} = e + \mathrm{Ker}\, a \in \bar{I}$ which is a modulus for $\bar{K}$. We can assume that $e \in I$, hence $ae \in I$ since I is an ideal, and

(iii) $I + L = R$

since L is ae-modular by (ii).

Altogether (i)–(iii) show that L is a primitizer (resp. $*$-primitizer) of R. □

For an idempotent e in an associative algebra R, the Peirce (1,1)-component eRe is readily seen to be isomorphic to the local algebra R_e. This, together with (1.1), yields the following result on inheritance of primitivity from the Peirce (1,1)-components.

1.2. Corollary. *Let R be a prime (resp. $*$-prime) associative algebra (resp. $*$-algebra) such that eRe is left (right) primitive (resp $*$-primitive) for some idempotent $0 \neq e \in R$ (resp. $0 \neq e \in H(R,*)$). Then R is primitive (resp. $*$-primitive).*

2. Non-homotope-PI Jordan algebras. We will begin with an auxiliary result on the relationship between the speciality of a Jordan algebra and its speciality as a Jordan triple system.

2.1. Lemma. *If a unital Jordan algebra J is special as a triple system, then it is special as an algebra. Moreover, there exists a unital (algebra) envelope and a unital $*$-envelope of J with the same unit element as J.*

Proof. Assume that J is a subtriple of $R^{(+)}$, where R is an associative triple system. By [2, 1.13], there exists an associative algebra A such that R is a subtriple of the underlying triple system of A. Thus J is a subtriple of $A^{(+)}$ (this latter considered as a triple system), i.e.,

(i) $$U_x y = xyx$$

for any $x, y \in J$, where juxtaposition denotes the product in A. In general, one cannot prove that $x^2 = xx$ in this situation. Indeed, this may be false, as shown when A is a unital associative algebra and we consider a subalgebra J of $(A^{(-1)})^{(+)}$ which then has $x^2 = x \cdot_{-1} x = x(-1)x = -xx$ and unit -1, though it is a subtriple of $A^{(+)}$ since $U_x y = x(-1)y(-1)x = xyx$. This example provides a suitable substitute for the algebra A we are dealing with:

Let b the unit element of J. Now, J is a subalgebra of $B^{(+)}$, where B is the b-homotope $A^{(b)}$ of A, which is an associative algebra, which shows that J is special:

$$x^2 = U_x b \text{ [since } b \text{ is the unit element of } J] = xbx \text{ [by (i)] } = x \cdot_b x$$

and

$$U_x y = U_x U_b y \text{ [since } b \text{ is the unit element of } J] = xbybx \text{ [by (i)]} = x \cdot_b y \cdot_b x.$$

Replacing A by the subalgebra of B generated by J, we have that A is an envelope of J, hence, with this change of notation, we also have, for any $x \in J$,

(ii) $\qquad x^2 = xx.$

Therefore

$$\begin{aligned} bx &= b(U_b x) \text{ [since } b \text{ is the unit element of } J] = bbxb \text{ [by (i)]} \\ &= b^2 xb \text{ [by (ii)]} = bxb \text{ [since } b \text{ is the unit element of } J] = U_b x \text{ [by (i)]} = x \end{aligned}$$

since b is the unit element of J. Similarly $xb = x$, for any $x \in J$. Using the fact that A is generated as an algebra by J the above equalities can be extended to $by = y = yb$ for any $y \in A$ showing that b is the unit element of A.

Finally, J can be considered as a subalgebra of $H(C, *)$ where $C = A \oplus A^{\mathrm{op}}$, A^{op} being the opposite algebra of A, and $*$ is the exchange involution. The new algebra C still has the same unit element as J and the subalgebra of C generated by J is the desired $*$-envelope with the same unit element as J. $\square$

The proof of local-to-global inheritance of primitivity for strongly prime non-homotope-PI Jordan algebras begins with a lemma in which the case of ample subspaces of associative algebras with involution is studied. In the general case, ideals of this form will be found, so that the general result will follow from an argument on inheritance of primitivity from ideals [8, 3.2].

2.2. Lemma. *Let J be a strongly prime Jordan algebra which is an ample subspace $H_0(R, *)$ of an associative algebra R with involution $*$, assume that J_a is primitive for some $0 \neq a \in J$. Then J is primitive.*

Proof. By factoring out a maximal $*$-ideal of R not hitting J, we can assume that R is $*$-tight over J, hence $*$-prime by strong primeness of J.

It can be readily seen that J_a is isomorphic to an ample subspace $H_0(R_a, *)$ of the local algebra R_a (cf. [4, 0.5]), which is $*$-prime by (0.7) (I) (ii). Thus R_a is $*$-primitive by [8, 4.9]. Now, R is $*$-primitive by (1.1), and $J = H_0(R, *)$ is primitive again by [8, 4.9]. $\square$

2.3. We will say that a Jordan algebra J is *homotope-PI* if there is a polynomial identity which is satisfied by all of its homotopes. Since homotopes just depend on the triple system structure of J (see (0.6)), a homotope-PI Jordan algebra J is just an algebra whose underlying triple system is homotope-PI (cf. [3, p. 212]).

2.4. Hearty eaters. Recall [1, 3.7] that an adic family on a special Jordan triple system J is a family of n-linear maps $F_n : J^n \longrightarrow V$ into some Φ-module V for all odd $n \geqq 1$ satisfying

(T1) $F_n(\ldots, x, y, x, \ldots) = F_{n-2}(\ldots, P_x y, \ldots)$

and which preserves all pentads $\{x_1 x_2 x_3 x_4 x_5\} = x_1 x_2 x_3 x_4 x_5 + x_5 x_4 x_3 x_2 x_1$, $x_i \in J$, which fall back in J:

(T2) $F_n(\ldots, x_1, x_2, x_3, x_4, x_5, \ldots) + F_n(\ldots, x_5, x_4, x_3, x_2, x_1, \ldots) = F_{n-4}(\ldots, \{x_1 x_2 x_3 x_4 x_5\}, \ldots)$.

In [1, 3.12, 3.16, 4.5], D'Amour shows the existence of a nonzero ideal $\mathscr{H}_5(X)$ of the free special Jordan triple system $\mathrm{ST}(X)$ on an infinite set of variables X which eats pentads of arbitrary adic families on $\mathrm{ST}(X)$: For such an arbitrary adic family $\{F_n\}_{n \text{ odd}}$, $p \in \mathscr{H}_5(X)$ and variables $y_1, \ldots, y_4 \in X$ not appearing in p,

$$\text{(i)} \qquad F_5(y_1, \ldots, p, \ldots, y_4) = \sum_{j=1}^{m_p} F_3(q_{j1}, q_{j2}, q_{j3}),$$

where $q_{j1}, q_{j2}, q_{j3} \in \mathrm{ST}\,(X)$, m_p is a non-negative integer. We remark that p eats pentads from any position so that, indeed, five different equalities (i) hold, corresponding to the five different possibilities for the location of p as an argument of F_5 (that is the meaning of $F_5(y_1, \ldots, p, \ldots, y_4)$). It is shown that $\mathscr{H}_5(X)$ is a hermitian ideal in the sense [1, 1.2] that it is n-tad closed for all odd $n \geqq 5$:

$$\text{(ii)} \qquad \{\overbrace{\mathscr{H}_5(X) \ldots \mathscr{H}_5(X)}^{n}\} \subseteqq \mathscr{H}_5(X),$$

where $\{x_1 \ldots x_n\}$ denotes the associative polynomial $x_1 \ldots x_n + x_n \ldots x_1$, called an n-tad. From the construction of a nonzero element of $\mathscr{H}_5(X)$ [1, 4.5], it follows that $\mathscr{H}_5(X)$ contains homotope polynomials, i.e., if a special Jordan triple system J is non-homotope-PI, then the evaluation $\mathscr{H}_5(J)$ is nonzero [4, 2.2].

If J is a special Jordan algebra, its underlying triple system is special, so that $\mathscr{H}_5(J)$ is an ideal of J as a triple system. The next lemma is devoted to studying $\mathscr{H}_5(J)$ when J is unital.

2.5. Lemma. *Let J be a unital special Jordan algebra, R be a $*$-envelope of J with the same unit* 1 *as J, and $I = \mathscr{H}_5(J)$. Then I is an ideal of J which is n-tad closed:*

$$\{\overbrace{I \ldots I}^{n}\} \subseteqq I.$$

for all $n \geqq 4$.

Proof. From (2.4), I is an ideal of J as a triple system. Thus I is an ideal of the algebra J by (0.2) since J is unital. By (2.4), I is n-tad closed for all odd $n \geqq 5$. We will show that I is n-tad closed for all even $n \geqq 4$. Since 1 is the unit element in both J and R,

$$\begin{aligned} \{\overbrace{I \ldots I}^{n}\} = \{1 \overbrace{I \ldots I}^{n}\} = \{1 I \ldots I \overbrace{IIIII}^{\#}\} &\subseteqq \{1 I \ldots I \overbrace{JJJI\mathscr{H}_5(J)}^{\#}\} \\ &\subseteqq \{1 I \ldots I \overbrace{IJJ}^{\#}\} + \{1 I \ldots I \overbrace{JIJ}^{\#}\} + \{1 I \ldots I \overbrace{JJI}^{\#}\} \end{aligned}$$

by considering the adic family of imbedded n-tads when the variables not included in $\#$ are fixed, and using the specialization of (2.4) (i) when one of the variables y_i is evaluated in the ideal I of J, by homogeneity as in [1, 2.4]. The argument can be repeated to obtain that

$$\{\overbrace{I \ldots I}^{n}\} \subseteqq \{JJI\} + \{JIJ\} + \{JJI\} \subseteqq I$$

since I is an ideal of J. □

The next result will allow us to move the element at which the local algebra is considered.

2.6. Lemma. *Let J be a strongly prime Jordan algebra such that J_a is primitive for some $0 \neq a \in J$, and let I be a nonzero ideal of J. Then there exists $0 \neq a' \in I$ such that $J_{a'}$ is primitive.*

Proof. Notice that $U_a I \neq 0$. Otherwise, a lies in the annihilator of I by [14, 1.7(i)], hence $I = 0$ by strong primeness of J [14, 1.6]. Thus there exists $0 \neq a' = U_a b$ for some $b \in J$.

Recall the isomorphism $J_{a'} = J_{U_a b} \cong (J_a)_{b+\mathrm{Ker}_J a}$ given by

$$x + \mathrm{Ker}_J(U_a b) \longrightarrow (x + \mathrm{Ker}_J a) + \mathrm{Ker}_{J_a}(b + \mathrm{Ker}_J a)$$

(cf. [4, 4.3 (ii)]). Since $b \notin \mathrm{Ker}_J a$, $0 \neq b + \mathrm{Ker}_J a \in J_a$, and $(J_a)_{b+\mathrm{Ker}_J a}$ is primitive by (0.7) (II) (iii), which implies primitivity of $J_{a'}$ by the previous isomorphism. □

2.7. Proposition. *Let J be a unital, non-homotope-PI, strongly prime Jordan algebra such that J_a is primitive for some $0 \neq a \in J$. Then J is primitive.*

Proof. Notice that J is strongly prime as a triple system (see (0.2)). By [19; 2, 4.1], J is special as a triple system. Therefore, using (2.1), J is a special Jordan algebra, and there exists an associative $*$-envelope R of J with the same unit element as J. By (2.4) and (2.5) $I = \mathscr{H}_5(J)$ is a nonzero ideal of the algebra J which is n-tad closed. By [16, 1.3], I is an ample subspace of $H(B, *)$, where B is the subalgebra of R generated by I. Moreover, I is strongly prime by [14, 2.5], since it is an ideal of J and J is strongly prime.

On the other hand, by (2.6), there exists $0 \neq a' \in I$ such that $J_{a'}$ is primitive. Since $I_{a'}$ is isomorphic to a nonzero ideal of $J_{a'}$, $I_{a'}$ is primitive by [8, 3.1], hence I is primitive by (2.2), and J is primitive by [8, 3.2]. □

3. Homotope-PI Jordan algebras. The case of homotope-PI Jordan algebras will follow by using the Peirce decomposition and the socle. Again, to compare Jordan algebras with their underlying triple systems, we will restrict ourselves to unital algebras.

3.1. Proposition. *Let J be a unital, homotope-PI, strongly prime Jordan algebra such that J_a is primitive for some $0 \neq a \in J$. Then J is primitive.*

Proof. Recall that J is strongly prime as a triple system by (0.2), hence J is a primitive triple system by [4, 3.8] and has nonzero socle in the triple sense by [3, 6.2] since it is homotope-PI. But inner ideals in the triple sense and in the algebra sense coincide when there is a unit element, so that J has a nonzero socle as a Jordan algebra. By (2.6), we can assume that a lies in the socle of J, and there exists an isotope of $J^{(u)}$ of J (for an invertible

$u \in J$) where a is an idempotent [10, Lemma 6]. Since primitivity and strong primeness of J and $J^{(u)}$ are equivalent (ideals of J and $J^{(u)}$ coincide, so that a primitizer of J is a primitizer for $J^{(u)}$), and $(J^{(u)})_a \cong J_a$ [9, 3.2], we can assume that $a = e$ is an idempotent of J.

Now, J_e is a primitive PI Jordan algebra since J is homotope-PI, hence J_e is simple by [8, 1.2], but, by [9, 3.3], J_e is isomorphic to the Peirce component U_eJ of J, so that U_eJ is a simple algebra.

We will show that the inner ideal $K = U_{1-e}J$ is a primitizer of J. Clearly K is proper by nondegeneracy of J, since $0 \neq e$ and $U_eK = 0$, and it is 1-modular as any other inner ideal of a unital Jordan algebra (indeed, in this case K is also e-modular since e is an idempotent). Let I be a nonzero ideal of J. As in the proof of (2.6), $0 \neq U_eI \subseteqq I \cap U_eJ$, hence $I \cap U_eJ = U_eJ$ by simplicity of U_eJ, and $e = U_e e \in U_eJ \subseteqq I$. But

$$J = U_eJ + U_{e,1-e}J + U_{1-e}J = U_eJ + U_{e,1-e}J + K \subseteqq I + K$$

since $e \in I$ and I is an ideal of J, which shows that K complements nonzero ideals of J. □

4. Main results. To prove a Jordan version of (1.1), we just need to put together (2.7) and (3.1) and use tight unital hulls to extend the outcome to non-unital Jordan algebras:

4.1. Theorem. *Let J be a strongly prime Jordan algebra such that J_a is primitive for some $0 \neq a \in J$. Then J is primitive.*

Proof. Let $\hat{J}$ be a tight unital hull of J. By tightness, $\hat{J}$ is prime, while its nondegeneracy follows by [15, 2.9 (iii)]. It is readily seen that J_a is isomorphic to a nonzero ideal of $\hat{J}_a$, and $\hat{J}_a$ is strongly prime by (0.7) (II) (ii). Hence $\hat{J}_a$ is primitive by [8, 3.2], and $\hat{J}$ is primitive by either (2.7) or (3.1). Now J is primitive by [8, 3.1]. □

Using [9, 3.3], we obtain a Jordan version of (1.2):

4.2. Corollary. *Let J be a strongly prime Jordan algebra such that the Peirce component U_eJ of J, for a nonzero idempotent $e \in J$, is primitive. Then J is primitive.* □

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Eingegangen am 21. 4. 1997

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